A double complex for computing the sign-cohomology of the universal ordinary distribution

Greg W. Anderson

School of Mathematics
University of Minnesota
Minneapolis, MN 55455
U.S.A.

Abstract

The universal ordinary distribution \( U \) is defined to be

* the free abelian group on symbols of the form \([a]\) with \( a \in \mathbb{Q}/\mathbb{Z} \),
* modulo relations of the form \([a] - \sum_{nb=a} b\) with \( a \) as above and \( n \) a positive integer.

The basic results on the structure of \( U \) are due to Kubert. The group \( U \) comes equipped with an involution ("complex conjugation") induced at the symbol level by the map \([a] \mapsto [−a]\). We are interested in the cohomology of \( U \) with respect to the two-element group generated by complex conjugation ("sign-cohomology of the universal ordinary distribution"). These sign-cohomology groups were first computed by Kubert, building upon work Sinnott.

To provide background we begin the talk by explaining

* how the sign-cohomology groups first arose in Sinnott’s computation of the unit index.
* the basic results on the universal ordinary distribution and its sign-cohomology.
MacMahon’s Partition Analysis

George Andrews
Pennsylvania State University
Department of Mathematics
410 McAllister Building
University Park, PA 16802-6401

Abstract

In this talk we examine a combinatorial technique developed by P.A. MacMahon at the turn of the century: Partition Analysis. He devoted all of Section VIII in the second volume of his book, Combinatory Analysis, to this topic. Unfortunately, he was unable to apply this method successfully to his focus of interest at the time, plane partitions. This setback caused the method to fall into disuse. Only one other time this century did anyone use the method on a major problem; in the early 1970’s Richard Stanley solved the Anand-Dumir-Gupta Magic Squares Conjecture using partition analysis.

Now that we live in an age of advanced computer algebra systems, it is time to reconsider Partition Analysis. In this talk I shall outline the basic features of the method by applying it to some elementary problems. Then I shall discuss its applications to the recently discovered and magnificent Lecture Hall Partition Theorem of Bousquet-Melou and Eriksson. We shall also discuss the project of retrieving MacMahon’s original mission. Finally we shall look to the future.
Down-up Algebras

Georgia Benkart

Department of Mathematics, University of Wisconsin
480 Lincoln Dr, Madison
WI 53706-1388, USA

Abstract

The algebra generated by the down and up operators on a differential partially ordered set (poset) encodes many of the enumerative and structural properties of the poset. In this work we define a family of infinite-dimensional associative algebras, called down-up algebras, which are the universal enveloping algebras for the algebras of down and up operators on posets. We discuss the structure and representations of down-up algebras. They have a well-behaved highest weight theory which in some ways resembles that of the Lie algebra sl(2), but has many additional interesting features. The universal enveloping algebras of certain Lie algebras and some subalgebras of quantum groups are examples of down-up algebras, but the class of down-up algebras is much more general.

\[\text{1 The author gratefully acknowledges support from National Science Foundation Grant #DMS-9622447.}\]
Coherent and motivis Galois module structures

Ted Chinburg
University of Pennsylvania
Department of Mathematics
Philadelphia, PA 19104-6317

Abstract

Two classical problems of algebraic number theory are to describe the Galois module structure of the ring of integers and of the unit group of the top field of a finite Galois extension of number fields. In this talk I will give a survey of recent progress on generalizations of these problems to higher dimensional schemes and motives.

The first generalization concerns the Galois structure of the cohomology of coherent sheaves on projective schemes over the integers having a tame action by a finite group. This is joint work with G. Pappas, M. Taylor and B. Erez. One result shows that the equivariant Euler characteristic of a certain modified de Rham complex can be determined from root numbers. This generalizes Taylor’s proof of Frohlich’s conjecture concerning rings of integers. I will also discuss some recent work on higher dimensional Hermitian Galois structure, with connections to Arakelov theory.

The second generalization concerns the Galois structure of motivic cohomology. I will describe some joint work with G. Pappas, M. Kolster and V. Snaith, with specific applications to the Galois structure of K-groups and Mordell Weil groups.
Stable invariant vector bundles over modular curves $X(p)$

Igor V. Dolgachev

University of Michigan
Department of Mathematics
Ann Arbor, MI 48109-0001

Abstract

Let $X(p$) be the modular curve corresponding to the principal congruence subgroup $\Gamma(p)$ of the modular group. In this talk I shall discuss the problem of classification of stable vector bundles on $X(p)$ which are invariant with respect to the natural action of the group $G = PSL(2, F_p)$ on $X(p)$. We shall start with the case of line bundles where the result was known in the form of classification of automorphy factors for fuchsian groups. Then we proceed to the case of rank 2 bundles and show how this problem can be approached via the theory of unitary representation of the fundamental groups of homology 3-spheres. Applying this theory we shall prove that $X(p)$ admits exactly $2n$ non-isomorphic rank 2 invariant stable bundles with trivial determinant, where $p$ is a prime number of the form $6n \pm 1$. Then we shall consider the special cases $p = 7$ and 11 to discuss the beautiful geometry associated to these bundles. It goes back to Felix Klein and has been revived recently in the work of several mathematicians. We shall also give a geometric construction of four invariant stable bundles of rank 3 with trivial determinant over $X(7)$ and prove that there are no more.
Okubo algebras and twisted polynomials.

Alberto Elduque

Departamento de Matematicas y Computacion
Edificio Departamental
Universidad de La Rioja
26004 Logroño, Spain

Abstract

Okubo algebras form a class of non-unital composition algebras with interesting properties. Its complete classification over fields of characteristic not 3 was obtained jointly with H.C. Myung in 1933, by relating them to some central simple associative algebras of degree 3 with involution. Only this year the classification in characteristic 3 was completed by completely different arguments, which relate them to some polynomial algebras. The aim of the talk is to show that both cases are not so unrelated as it may look at first glance, since the situation that appears in characteristic 3 will be proven to be a kind of limit of the situation in other characteristics. This allows us to give a unified multiplication table for the Okubo algebras with isotropic quadratic form valid in any characteristic.
New results about modular forms for $GL(2, \mathbb{F}_q[T])$

E.-U. Gekeler
Fachbereich 9 Mathematik
Universität des Saarlandes
Postfach 15 11 50
D-66041 Saarbrücken

Abstract

It is well known that the ring $A = \mathbb{F}_q[T]$ and some derived data share many properties with $\mathbb{Z}$, the ring of integers, and the corresponding data. This holds in particular for the group $\Gamma = GL(2, A)$, for which a modular theory exists that in many respects parallels the classical theory of elliptic modular forms.

Modular forms for $\Gamma$ are holomorphic functions on the Drinfeld upper half-plane, a rigid analytic symmetric space. Basic examples of such modular forms are Eisenstein series (certain lattice sums) and coefficient forms associated to Drinfeld modules. Their theory is therefore linked to the geometry of Drinfeld modular schemes.

In the first part of the talk, we give a brief overview on known results on such "Drinfeld modular forms": structure of the algebra of modular forms, their expansions around $\infty$, the $j$-invariant, congruence properties, formulae for some specific forms.

In the second part, we present new results e.g. about the zeroes of Eisenstein series and the fields generated by their $j$-invariants. New questions about classical modular forms arise.
Property (τ) and its applications in combinatorics, geometry and group theory

Alex Lubotzky

Abstract

Kazhdan property (T) is an important property of semi-simple Lie groups and their arithmetic lattices. Property (τ) is a weaker form of property (T) which happens to have a number of equivalent forms in analysis, combinatorics, measure theory and geometry. We will present these forms and illustrate how this property can be used in a wide range of applications. Beginning with the construction of expander graphs which are of fundamental importance in computer science and ending with showing non-vanishing of Betti-number of some hyperbolic manifolds (which is a well known conjecture of Thurston).
Abstract

A prounipotent group is an inverse limit of an inverse system of unipotent algebraic groups with surjective algebraic transition maps (over the complex numbers). An example would be formal power series of complex $n$ by $n$ matrices with constant term $I$. A pronilpotent Lie algebra is an inverse limit of an inverse system of finite dimensional nilpotent Lie algebras with surjective transition maps. An example would be the Lie algebra of formal power series of complex $n$ by $n$ matrices with no constant term. Starting with any algebraic subgroup $G$ of $GL(n, C)$ one can consider $G(C[[t]])$ and its intersection $UG$ with the above prounipotent group and then ask how $G$ can be recovered from the prounipotent group $UG$. Similarly, one can start with a Lie subalgebra $L$ of $M(n, C)$, from the Lie algebra of formal power series with coefficients in $L$ and consider its intersection $NL$ with the above pronilpotent Lie algebra, and ask how $L$ can be recovered from $NL$. These problems can be related to construction problems in differential Galois theory.
Graded simple Jordan superalgebras of growth one

Consuelo Martinez
University of Oviedo, Spain

Abstract

Superconformal algebras are Lie superalgebras that are extensions of Virasoro algebras. Such superalgebras have a good representation theory and are of great interest in physics. A way to approach to their classification is through Jordan theory. With this aim we study Z-graded Jordan superalgebras that are graded simple and the dimensions of all homogeneous components are uniformly bounded and the classification theorem for such superalgebras is obtained.
Quantum Convolution

Earl J. Taft
Rutgers University
Department of Mathematics
New Brunswick, NJ 08903-2101

Abstract

We consider sequences \( f = (f_i), i \geq 0, f_i \) in a field \( k \), under the convolution product \( f * g = h \), where \( h_i = \sum_{j=0}^{i} \binom{i}{j} f_j g_{i-j} \). The Hopf dual \( k[x]^0 \) of the polynomial Hopf algebra \( k[x] \) with \( x \) primitive can be identified with the linearly recursive sequences \( f.i.e., f \) for some \( r > 0, f_i = h_1 f_{i-1} + h_2 f_{i-2} + \cdots + h_r f_{i-r} \) for all \( i \geq r \), for fixed field elements \( h_1, \ldots, h_r \). Let \( q \) be a non-zero element of \( k \). We consider the \( q \)-convolution product of sequences \( f \) \( *_q g = h \), where \( h_i = \sum_{j=0}^{i} \binom{i}{j}_q f_j g_{i-j} \), where \( \binom{i}{j}_q \) are the Gaussian polynomials. We show that the space of linearly recursive sequences is closed under \( q \)-convolution if and only if \( q \) is a root of unity. More precisely, if \( q \) is a primitive \( n \)-th root of unity, then \( k[x]^0 \) is a Hopf algebra in the braided monoidal category of modules over the quasi-triangular Hopf algebra \( kG \), where \( G \) is the cyclic group of order \( n \), but with a non-standard \( R \)-matrix giving the quasi-triangular structure of \( kG \), where \( G \) is the cyclic group of quasi-triangular structure of \( kG \). If \( q \) is not a root of unity, we show that the \( q \)-product of two non-zero geometric (exponential) sequences is not linearly recursive. There are also combinatorial aspects involving Rogers-Szegö polynomials.
Weil Classes on Abelian Varieties

Yuri G. Zarhin

Pennsylvania State University

Abstract

A Hodge class on a complex projective variety is called decomposable if it can be presented as a linear combination of products of divisor classes. A theorem of Lefschetz implies that up to dimension 3 all Hodge classes are decomposable.

About 30 years ago Mumford gave a first example of non-decomposable Hodge class. This class lives on an abelian fourfold of CM-type. A few years later, Weil noticed that the essential ingredient in Mumford’s example is the fact the endomorphism algebra of an even-dimensional abelian variety contains an imaginary quadratic subfield, which acts with equal multiplicities, consists of Hodge classes. Moreover, Weil showed that for "generic" even-dimensional abelian varieties with action by an imaginary quadratic field (and the condition on the multiplicities) all non-zero Weil classes are non-decomposable. Weil’s construction and its natural generalizations have numerous applications in (arithmetical) algebraic geometry.

In this talk (based on a join work with Ben Moonen) I give an explicit necessary and sufficient condition for (non-zero) Weil classes to be non-decomposable.
On the Burnside Problem and its relations

E. Zelmanov

Department of Mathematics, Yale University
Box 208283, New Haven, CT 06520-8283

Abstract